AS
Mathematics

MFP1 Further Pure 1
Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	DO NOT ALLOW ANY MISREADS IN THIS QUESTION			
	$h\,y'(4) = 0.3 \times \left(\frac{1}{8+\sqrt{4}}\right) \quad (= 0.03)$	**M1**		Attempt to find $h\,y'(4)$
	$\{y(4.3)\} = 8 + 0.03 = 8.03$	**A1**		8.03 OE
	$\{y(4.6)\} = y(4.3) + 0.3\,y'(4.3)$ $= 8.03 + 0.3 \times \left(\frac{1}{2(4.3)+\sqrt{4.3}}\right)$	**dM1**		Attempt to find $y(4.3) + 0.3\,y'(4.3)$; must see evidence of numerical expression if correct ft [0.0281…+c's y(4.3)] value is not obtained.
	$= 8.03 + 0.3 \times 0.09368....$ $= 8.03 + 0.0281....$	**A1F**		PI ft on c's value for y(4.3); 4dp (rounded or truncated) or better
	$y(4.6) = 8.0581$ (to 4dp)	**A1**	**5**	CAO Must be 8.0581 identified as y(4.6) or as c's final answer or as c's highlighted answer.
	Total		**5**	

Q2	Solution	Mark	Total	Comment
(a)	$\alpha+\alpha+4=-\dfrac{p}{5}$; $\alpha(\alpha+4)=\dfrac{q}{5}$	B1; B1		
	$\alpha+2=-\dfrac{p}{10}$; $(\alpha+2)^2=\dfrac{q}{5}+4$			
	$\dfrac{p^2}{100}=\dfrac{q}{5}+4$	M1		Eliminating α to form an eqn in p and q only, dep on at least B1 scored above. **M0** if >1 indep error in process before the line where α has been eliminated
	$p^2=100(\dfrac{q}{5}+4)\Rightarrow p^2=20q+400$	A1	4	AG Be convinced
Alt 1	$(x=)\dfrac{-p\pm\sqrt{p^2-20q}}{10}$	(B1)		PI
	Equating one correct root to α and the other correct root to $\alpha+4$	(B1)		PI
	$(\pm)4=\dfrac{2\sqrt{p^2-20q}}{10}$	(M1)		Eliminating α to form an eqn in p and q only, condone 1 sign error in roots of eqn
	$\sqrt{p^2-20q}=(\pm)20\Rightarrow p^2=20q+400$	(A1)	(4)	AG Be convinced
Alt 2	$5(\alpha+4)^2+p(\alpha+4)+q=0$ and $5\alpha^2+p\alpha+q=0$	(B1)		Both required if a B1 not scored from main scheme.
	Subtract eqns to get $\alpha=-2-0.1p$	(B1)		OE linear eqn in α and p only
	$5(-2-0.1p)^2+p(-2-0.1p)+q=0$	(M1)		Eliminating α to form an eqn in p and q only, condone 1 sign error in 2nd B mark
	$20-0.05p^2+q=0$ so $p^2=20q+400$	(A1)	(4)	AG Be convinced
(b)(i)	$S[=2(\alpha^2+4\alpha+8)]=2\left(\dfrac{q}{5}+8\right)$	B1		A correct expression for the sum of the new roots in terms of q only
	$P[=\alpha^2(\alpha+4)^2]=\left(\dfrac{q}{5}\right)^2$	B1		A correct expression for the product of the new roots in terms of q only
	$x^2-2\left(\dfrac{q}{5}+8\right)x+\left(\dfrac{q}{5}\right)^2=0$	B1F	3	Ft c's S and P to form a quadratic eqn in terms of q with no square roots.
Alt	Subst $y=x^2$ gives $5y+p\sqrt{y}+q=0$	(B1)		
	$p^2y=(-5y-q)^2$	(B1)		OE with no square root
	$25y^2-(10q+400)y+q^2=0$	(B1)	(3)	ACF of quadratic eqn in terms of q and the variable only with relevant terms grouped
(ii)	$4\left(\dfrac{q}{5}+8\right)^2=4\left(\dfrac{q}{5}\right)^2\Rightarrow\dfrac{16q}{5}+64=0$	M1		Use of $B^2-4AC=0$ OE to obtain a **linear** eqn in q.
	$q=-20$	A1	2	$q=-20$ NMS 2/2
(ii) Alt	$(\alpha+4)^2=\alpha^2\Rightarrow\alpha=-2$	(M1)		$\alpha=-2$
	$q=5(-4)=-20$	(A1)	(2)	$q=-20$ NMS 2/2
	Total		9	
	(b)(ii) Both marks can be scored without (b)(i) being correct.			

Q3	Solution	Mark	Total	Comment
(a)	$z = \mathrm{i}(1-\mathrm{i})(2+\mathrm{i}) = \mathrm{i}(2+\mathrm{i}-2\mathrm{i}-\mathrm{i}^2)$			
	$= 2\mathrm{i}-\mathrm{i}^2-\mathrm{i}^3$	M1		Attempt to expand all brackets
	$z = 2\mathrm{i}-(-1)-(-\mathrm{i})$	M1		$\mathrm{i}^2 = -1$ used at least once at any stage in part **(a)**
	$z = 1+3\mathrm{i}$	A1	3	$1+3\mathrm{i}$ obtained convincingly **SC 1** $1+3\mathrm{i}$ NMS
(b)	$z-\mathrm{i} = 1+2\mathrm{i}$	B1F		c's k + 2i. PI by next line
	$(z-\mathrm{i})^* = 1-2\mathrm{i}$	B1F		c's $k-2\mathrm{i}$
	$1-2\mathrm{i}-m(1+3\mathrm{i}) = n(1+4\mathrm{i})$ (#)			
	Re: $1-m=n$; Im: $-2-3m=4n$	M1		Attempting to equate, without mixing real and imaginary terms, both the Re parts **and** the Im parts to form two eqns each in m and n for the c's eqn (#).
	$-2-3m = 4(1-m)$	A1		A correct eqn in either m only or in n only PI by correct values for both m and n.
	$m=6,\quad n=-5$	A1	5	Both required, be convinced.
	Total		**8**	

Q4	Solution	Mark	Total	Comment
(a)	$\int \frac{1}{2x\sqrt{x}}\,\mathrm{d}x = \int \frac{1}{2}x^{-1.5}\,\mathrm{d}x$	B1		$\frac{1}{x\sqrt{x}} = x^{-\frac{3}{2}}$ seen or used (ignore errors in dealing with the coefficient ½)
	$= -x^{-0.5}$ (+ constant)	B1		$-x^{-0.5}$ OE Integration correct
	$\int_c^d \frac{1}{2x\sqrt{x}}\,\mathrm{d}x = -\frac{1}{\sqrt{d}}+\frac{1}{\sqrt{c}}$	B1	3	OE
(b) (i)	$\frac{1}{\sqrt{c}} \to \infty$ as $c \to 0^{(+)}$ so integral has no finite value	E1		OE Ft on kc^{-n}, $n>0$ after integration
(ii)	$\frac{1}{\sqrt{d}} \to 0$ as $d \to \infty$	M1		OE Ft on kd^{-n}, $n>0$ after integration
	so $\int_9^\infty \frac{1}{2x\sqrt{x}}\,\mathrm{d}x = \frac{1}{3}$	A1	3	
	Total		**6**	
(b)(i)(ii)	Do NOT allow examples where c=0 eg $\frac{1}{\sqrt{0}} \to \infty$ or where $d=\infty$ eg $\frac{1}{\sqrt{\infty}} \to 0$			
(b)(i)(ii)	If 0/3 SC1 if in (i) after integration cand has kx^{-n}, n>0 then eg '$c \to 0$, so no finite value' or eg '$c \to 0$, so 'undefined''			

Q5	Solution	Mark	Total	Comment
(a)	$\sqrt{3}=\tan\frac{\pi}{3}$	B1		$\sqrt{3}=\tan\frac{\pi}{3}$ OE stated or used.
	$\left(2x+\frac{\pi}{2}\right)=n\pi+\frac{\pi}{3}$	M1		Ft c's $\tan^{-1}\sqrt{3}$. Condone 180n in place of $n\pi$
	$x=\frac{n\pi}{2}-\frac{\pi}{4}+\frac{\pi}{6}$	A1F		Ft c's $\tan^{-1}\sqrt{3}$. No degrees present
	$x=\frac{n\pi}{2}-\frac{\pi}{12}$	A1	4	OE form with constant terms combined
(b)	$\sin 4x=\sin\left(2n\pi-\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$	B1		Must be from correct GS
	$\sin 3x=\sin\left(\frac{3n\pi}{2}-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$	B1		OE exact values; need both. Must be from correct GS
	$\sin 3x-\sin 4x=\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}$ and $\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}$	B1	3	OE exact forms; need both **SC** if 0/3 scored award 1 mark if (i) cand gets (2 possible values for sin3x and only one possible value for sin4x) or (ii) cand obtains the two correct exact values by just considering specific values of n in the correct GS **NMS** Mark as 1/3 max.
	Total		7	
Altn (a)	Those using $2n\pi$, must be considering separately an angle in 1st quadrant and an angle in 3rd quadrant eg $\left(2x+\frac{\pi}{2}\right)=2n\pi+\frac{\pi}{3}$ and $\left(2x+\frac{\pi}{2}\right)=2n\pi+\frac{4\pi}{3}$ OE before M1 can be awarded eg $\sqrt{3}=\tan\left(\pm\frac{\pi}{3}\right)$ allow B1 only.			

Q6	Solution	Mark	Total	Comment
(a)	Vertical tangents: $x=4,\quad x=-4$ Horizontal tangents: $y=2,\quad y=-2$	M1		Identification of the tangents either stated or shown on a diagram. PI by correct area.
	Area of rectangle = $8\times 4=32$	A1	2	32 NMS 2/2
(b)	$\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$	B2,1	2	B2 else B1 for $\begin{bmatrix}1 & 0\\0 & k\end{bmatrix}$, $k\neq 0,\ k\neq 1$
(c) (i)	Translation maps $(4,0)$ to $(7,*)$ and $(-4,0)$ to $(-1,*)$	M1		Either pair; or 'statement indicating move 3 to the right'. PI by correct value for *a*.
	$\Rightarrow a=3$	A1	2	Correct value for *a*.
(c) (ii)	$E_2:\ \dfrac{(x-a)^2}{16}+\dfrac{(y-b)^2}{4}=1$ $4(x-a)^2+16(y-b)^2=64$	M1		Eliminating denominators to get $4(x-a)^2+16(y-b)^2=64$ OE seen or used. PI by $p=-2a$ **and** either $q=-8b$ or $16-a^2-4b^2=3$
	$x^2+4y^2-2ax-8by=16-a^2-4b^2$ Compare with $x^2+4y^2+px+qy=3$ $\Rightarrow p=-2a\qquad \Rightarrow p=-6$	B1		Correct value for *p*. Accept either from comparing with $(x-3)^2$ or with $(x-a)^2$
	Comparing coefficients of *y* and constant terms: $q=-8b;\quad 16-a^2-4b^2=3$	M1		OE **Both** attempted with at least one correct **or** $3+\dfrac{p^2}{4}+\dfrac{q^2}{16}=16$ OE
	$\Rightarrow b^2=1\quad \Rightarrow b=\pm 1\quad \Rightarrow q=\pm 8$	A1	4	Correct values for *q*.
	Total		10	
(c)(ii) Alt for M1	(Translate E_2 onto E_1 using translation $\begin{bmatrix}-a\\-b\end{bmatrix}$): $(x+a)^2+4(y+b)^2+p(x+a)+q(y+b)=3$ seen or used (**M1**) PI by $p=-2a$ **and** either $q=-8b$ or $16-a^2-4b^2=3$			

Q7	Solution	Mark	Total	Comment
(a)	$\sum_{r=1}^{n}(r^3-3r)= \sum_{r=1}^{n}r^3 - 3\sum_{r=1}^{n}r$	**M1**		$\sum_{r=1}^{n}(r^3+\beta r)=\sum_{r=1}^{n}r^3+\beta\sum_{r=1}^{n}r$ seen/used.
	$=\frac{n^2}{4}(n+1)^2-3\frac{n}{2}(n+1)$	**dM1**		Substitution of correct expressions for $\sum_{1}^{n}r^3$ and $\sum_{1}^{n}r$
	$=\frac{n}{4}(n+1)[n(n+1)-6]$	**dM1**		Taking out factor $n(n+1)$ or other product of 2 factors in *n* from the correct expression $\frac{1}{4}(n^4+2n^3-5n^2-6n)$
	$=\frac{n}{4}(n+1)[n^2+n-6]$			
	$=\frac{n}{4}(n+1)(n+3)(n-2)$	**A1**	**4**	$\frac{n}{4}(n+1)(n+3)(n-2)$ convincingly obtained
(b)	Series = $1^2+2^2+3^2+4^2+...+(2n)^2$ $-2[2^2+4^2+...+(2n)^2]$	**M1**		PI by the next line in soln
	$=\sum_{r=1}^{2n}r^2-8\sum_{r=1}^{n}r^2$	**A1**		PI by the next line in soln
	$=\frac{2n}{6}(2n+1)(4n+1)-8\frac{n}{6}(n+1)(2n+1)$	**B1**		$\sum_{r=1}^{2n}r^2=\frac{2n}{6}(2n+1)[2(2n)+1]$ or better
	$=\frac{2n}{6}(2n+1)[4n+1-4(n+1)]$			
	$=-n(2n+1)$	**A1**	**4**	$-n(2n+1)$ convincingly obtained
Alt (b)	Series = $1^2+3^2+5^2+...+(2n-1)^2$ $-[2^2+4^2+...+(2n)^2]$	**(M1)**		PI by the next line in soln, but must see difference between two series
	$=\sum_{r=1}^{n}(2r-1)^2-\sum_{r=1}^{n}(2r)^2$			
	$=\sum_{r=1}^{n}(-4r+1)=-4\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$	**(A1)**		$-4\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$ PI by the next line in soln
	$=-4\frac{n}{2}(n+1)+n$	**(B1)**		$\sum_{r=1}^{n}1=n$ seen or used
	$=-n(2n+1)$	**(A1)**	**(4)**	$-n(2n+1)$ convincingly obtained
	Total		**8**	
(b)	$(2n-1)^2-(2n)^2=-4n+1=-4(n/2)(n+1)+n$ scores **M0 B0** as no difference between 2 series			

Q8	Solution	Mark	Total	Comment
(a)	**D** = $\begin{bmatrix} 1 & 2.5 \\ 3.5 & -1 \end{bmatrix}$	**B2,1**	**2**	If not **B2** award **B1** for either (i) 3 elements correct or (ii) 2**D** = $\begin{bmatrix} 2 & 5 \\ 7 & -2 \end{bmatrix}$ seen or (iii) -2**D** = $\begin{bmatrix} -2 & -5 \\ -7 & 2 \end{bmatrix}$ seen or (iv) **D** = $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix}$ seen
(b)	Reflection in the line $y = -x$.	**E1**	**1**	OE eg $y = x\tan 135^{(\circ)}$
(c)(i)	$\cos\theta = -\frac{4}{5}$	**B1**		seen or used
	B = $\begin{bmatrix} -\frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$	**B1F**	**2**	Ft only on wrong sign for $\cos\theta$. Values must be exact
(ii)	**BA** = $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$ or **A**$\begin{bmatrix} 10 \\ 15 \end{bmatrix} = \begin{bmatrix} -15 \\ -10 \end{bmatrix}$	**M1**		Seen or used. Condone one arithmetical slip in evaluating the product of correct matrices.
	BA$\begin{bmatrix} 10 \\ 15 \end{bmatrix} = \begin{bmatrix} 18 \\ -1 \end{bmatrix}$ or **B** $\begin{bmatrix} -15 \\ -10 \end{bmatrix} = \begin{bmatrix} 18 \\ -1 \end{bmatrix}$	**A1**		At least one element of $\begin{bmatrix} 18 \\ -1 \end{bmatrix}$ correct, and correctly obtained.
	(*P* has coordinates) $(18, -1)$	**A1**	**3**	$(18, -1)$ **SC** If 0/3 award 1 mark for either $\begin{bmatrix} -6 \\ -17 \end{bmatrix}$ or $\begin{bmatrix} 6 \\ 17 \end{bmatrix}$ in matrix or coordinate form
	Total		**8**	
	SC in (c)(ii) $(-6,-17)$ from wrong sign for $\cos\theta$ and $(6,17)$ from using **AB** instead of **BA**.			

Q9	Solution	Mark	Total	Comment
(a)	$x=-1;\ \ x=3;\ \ y=2$	B2,1,0	2	OE . Each must be an **equation**. **B1** for two correct equations and no more than one incorrect equation.
(b)	$k=\dfrac{2x^2+2x+1}{(x+1)(x-3)}$	M1		Elimination of *y* to form an equation in *k* and *x*. Condone one sign error if the denominator has been expanded.
	$k(x^2-2x-3)=2x^2+2x+1$			
	$(k-2)x^2-2(k+1)x-(3k+1)=0$ **(*)**	A1		OE in form $ax^2+bx+c=0$
	$y=k$ intersects C so roots of (*) are real			
	$b^2-4ac=4(k+1)^2-4(k-2)(-3k-1)$	M1		b^2-4ac in terms of *k*; ft on c's quadratic provided *a* , *b* and *c* are all in terms of *k*.
	$4(k+1)^2-4(k-2)(-3k-1)\geq 0$	A1		A correct inequality obtained correctly where *k* is the only unknown
	$k^2+2k+1+3k^2-5k-2\geq 0$			
	$4k^2-3k-1\geq 0$	A1	5	CSO AG Be convinced
(c)	$(4k+1)(k-1)\geq 0$ (**)	M1		Method to find critical values from printed inequality in **(b)**. Condone one sign error. PI by correct two critical values
	Critical values are -0.25 and 1	A1		
	Sub $k=-0.25$ in (*), $9x^2+6x+1=0$ OE Sub $k=1$ in (*) gives $x^2+4x+4=0$ OE	dM1		Subst of either −0.25 or 1 into quadratic eq to reach a quadratic in *x* with equal roots
	$k=-0.25$, $x=-\dfrac{1}{3}$; $\left(-\dfrac{1}{3},-\dfrac{1}{4}\right)$ is a stationary point	A1		Correct corresponding values for *k* and *x* or correct coordinates
	$k=1$, $x=-2$; (−2, 1) is a stationary point	A1		Correct corresponding values for *k* and *x* or correct coordinates
	$PQ^2=\left(-\dfrac{1}{3}+2\right)^2+\left(-\dfrac{1}{4}-1\right)^2$	dM1		OE A correct numerical expression for either PQ^2 or PQ . Ft on c's wrong *x* values
	$PQ=\dfrac{25}{12}$	A1		ACF provided answer is exact value. ISW if $\dfrac{25}{12}$ is followed by a decimal.
			7	NMS scores 0/7; Using differentiation scores 0/7
	Total		**14**	